

100 points total (30 pts Part I & 70 pts Part II)

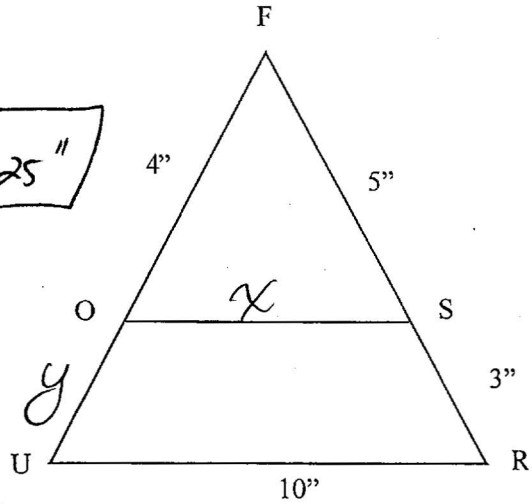
Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

1. (8 pts) Given the figure shown below with $OS \parallel UR$, find the following:

A. OS

$$\frac{FS}{OS} = \frac{FR}{UR}$$

$$OS = \frac{FS \cdot UR}{FR} = \frac{5(10)}{8} = \frac{50}{8} = \boxed{6.25''}$$



B. OU $\frac{OU}{OF} = \frac{SR}{FR}$

$$\frac{y}{4} = \frac{3}{5} \quad y = \frac{3(4)}{5} = \frac{12}{5} = \boxed{2.4''}$$

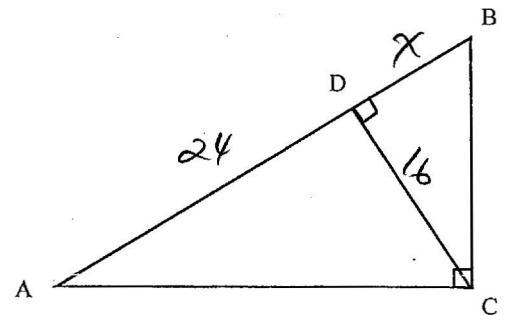
2. (6 pts) In $\triangle ABC$, if $AD = 24$ and $CD = 16$, find BD .

$$\frac{BD}{CD} = \frac{CD}{AD}$$

$$\frac{x}{16} = \frac{16}{24}$$

$$\frac{24x}{24} = \frac{(16)^2}{24} = \frac{256}{24}$$

$$x = \frac{256}{24} = \boxed{10.67 = \frac{32}{3}}$$



3.(8 pts) Given $\triangle ABC$ shown below. Find the exact length of the missing side. Also find $m\angle A$.

$$x^2 + 5^2 = 8^2$$

$$x^2 + 25 = 64$$

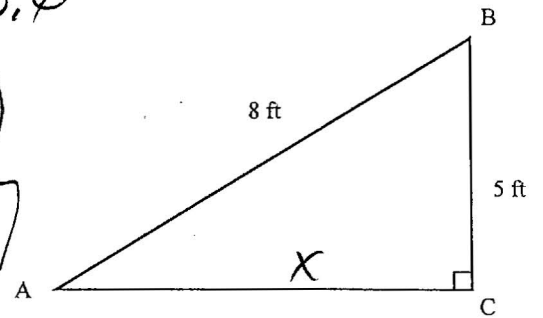
$$x^2 = 39$$

$$x = \sqrt{39} \text{ FT}$$

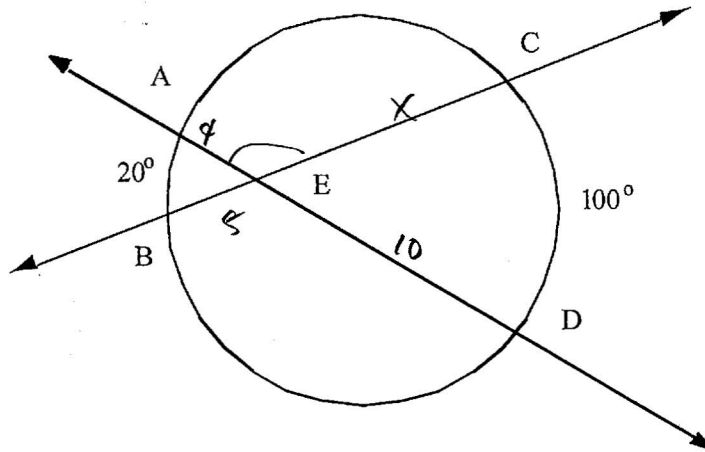
$$\sin A = \frac{5}{8} = 0.4$$

$$m\angle A = \sin^{-1}\left(\frac{5}{8}\right)$$

$$m\angle A = 38.68^\circ$$



4.(8 pts) Use the circle and secants to answer the following.



A. What is the measure of $\angle AEC$?

$$m\angle AEC = \frac{1}{2}(100 + 20) = \frac{1}{2}(120) = 60^\circ$$

$$m\angle AEC = 180 - 60 = \boxed{120^\circ}$$

B. If $AE = 4''$, $DE = 10''$, and $BE = 5''$, find CE .

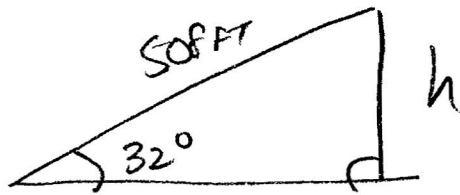
$$(AE)(DE) = (BE)(CE)$$

$$CE = \frac{(AE)(DE)}{BE}$$

$$= \frac{4(10)}{5} = 8''$$

$$\boxed{CE = 8''}$$

5.(8 pts) An escalator is 508 feet long and the angle it forms with the horizontal is 32° . What is the vertical distance traveled if a passenger rides from the bottom to the top of the escalator? Round to the nearest tenth.



$$\sin 32 = \frac{h}{508}$$

$$h = 508 \sin 32^\circ$$

$$h = 269.1989$$

$$\boxed{h = 269.2 \text{ FT}}$$

6.(8 pts) Suppose $\triangle ABC \sim \triangle DEF$, $AB = 5 \text{ cm}$, $BC = 9 \text{ cm}$, and $DE = 35 \text{ cm}$. Find EF .

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{EF}{DE} = \frac{BC}{AC}$$

$$EF = \frac{BC (DE)}{AC} = \frac{(9)(35)}{5} = \boxed{63 \text{ cm}}$$

7.(8 pts) Points A, B, and C are on circle O, as shown. $AC = 18 \text{ inches}$ and $m\angle AOB = 140^\circ$.

A. Find BC .

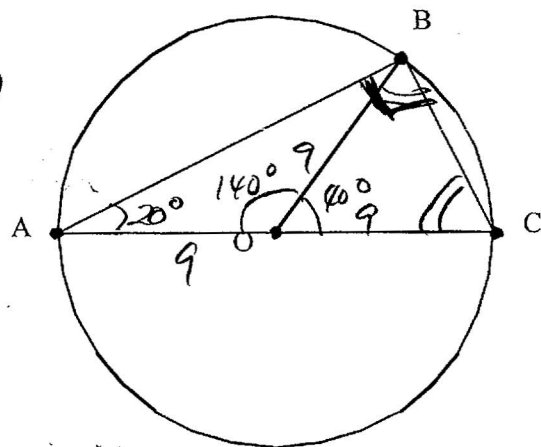
$$\sin A = \frac{BC}{AC}$$

$$BC = AC \sin A$$

$$= 18 \sin(20^\circ) = \boxed{6.16''}$$

B. Find $m\angle OBC$.

$$m\angle OBC = \frac{140}{2} = \boxed{70^\circ}$$



8. (16 pts) Given the figure shown below. Points E and F trisect diameter AC . Suppose $AC = 18$ inches.

Find:

A. Find the exact length of BC .

$$\frac{AE}{BE} = \frac{BE}{CE}$$

$$BE^2 = AE \cdot CE$$

$$BE^2 = 6(12)$$

$$BE^2 = 72$$

$$BE = 6\sqrt{2}''$$

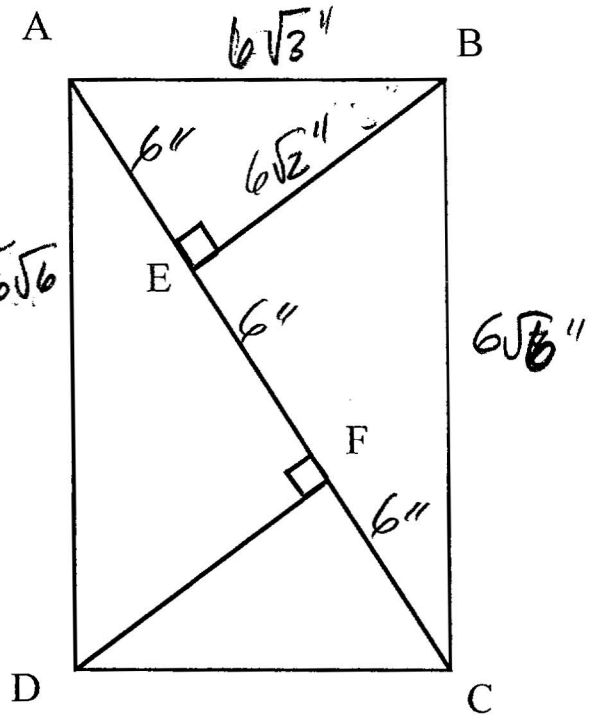
$$BE^2 + EC^2 = BC^2$$

$$(6\sqrt{2})^2 + 12^2 = BC^2$$

$$72 + 144 = BC^2$$

$$BC^2 = 216$$

$$BC = \sqrt{216} = \sqrt{36 \cdot 6} = 6\sqrt{6}''$$



B. Find the exact length of AB .

$$AB^2 = AE^2 + BE^2$$

$$= 6^2 + (6\sqrt{2})^2$$

$$= 36 + 72 = 108$$

$$AB = \sqrt{108} = 6\sqrt{3}''$$

C. Find the exact ratio of $\frac{BC}{AB}$.

$$\frac{6\sqrt{6}}{6\sqrt{3}} = \sqrt{2}$$

D. Find the area of the rectangle $ABCD$ to the nearest square inch.

$$\text{Area } ABCD = (6\sqrt{3})(6\sqrt{6}) = 36\sqrt{18} = 108\sqrt{2}$$

$$= 152.735 \text{ in}^2$$

$$\approx 153 \text{ in}^2$$



BONUS (total of 10 extra points)

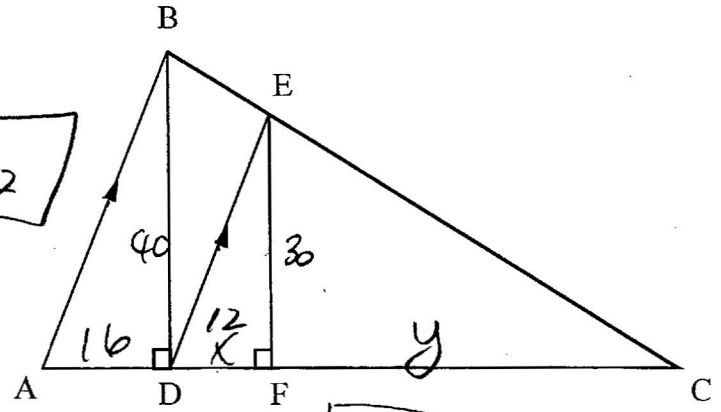


In $\triangle ABC$, $BD \perp AC$, $EF \perp AC$, and $AB \parallel DE$. $BD = 40$, $AD = 16$, and $EF = 30$.

Find the following:

$$\frac{DF}{EF} = \frac{AD}{BD}$$

$$DF = \frac{EF \cdot AD}{BD} = \frac{(30)(16)}{40} = \boxed{12}$$



$$\frac{DF}{EF} = \frac{CF}{EF}$$

$$\frac{CF}{EF} = \frac{EF}{DF}$$

$$CF = \frac{EF^2}{DF} = \frac{30^2}{12} = \boxed{75}$$

$$\frac{AB}{DE} = \frac{BD}{EF} = \frac{40}{30} = \boxed{\frac{4}{3} \approx 1.333}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} (AD + DF + FC) (BD) = \frac{1}{2} (16 + 12 + 75) (40)$$

$$= \boxed{2060}$$

$$\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle DEF} = \frac{\frac{1}{2} (AD)(BD)}{\frac{1}{2} (DF)(EF)} = \frac{(AD)(BD)}{(DF)(EF)} = \frac{(16)(40)}{(12)(30)} = \left(\frac{4}{3}\right)^2$$

$$= \boxed{\frac{16}{9} \approx 1.78}$$